Fractals, Chaos, Power Laws

By Manfred Schroeder, New York: Freeman, 1991, $32.95, 429 pp.

I cannot imagine anybody not learning something from this book. It is a *tour de force* covering an amazingly broad range of topics. In a rough sense, it can be thought as being in the tradition of recent popular books written by first-class scientists and mathematicians: D. R. Hofstadter, B. Mandelbrot, R. Penrose, and, if one is willing to include even more popular accounts, J. Gleick's *Chaos*. It, however, is quite different from any of these. It is less philosophical than Penrose's *The Emperor's New Mind* and Hofstadter's *Gödel, Escher, Bach*; it does not suddenly burst onto the stage unveiling a new paradigm as Mandelbrot's *The Fractal Geometry of Nature*.

The author, Manfred Schroeder, is a physicist who divides his time between Göttingen and AT&T Bell Laboratories and, has collected an incredible amount of information. The book covers an enormous array of topics, many of which are of direct relevance to people in chemical engineering. There is enough to keep the interest of people in: control (Markov processes, cellular automata, and noises—white and pink (1/f), as well as brown and black); physical chemistry and thermodynamics (phase transitions, the Ising model, renormalization, Brownian motion); transport (fractals, percolation, finite size scaling, diffusion-limited aggregation, DLA); fluid mechanics (multifractals and turbulence); and materials science (quasicrystals, self-similar agglomerates, percolation, for its obvious connection to gelation, fracture surfaces, diffraction in fractals, etc.). The closest it comes to chemistry is quantum mechanics and an example of a catalytic oxidation process in a catalytic converter (in the context of cellular automata). It, however, deals also with acoustics, music, psychology, economics, information theory, and even brief incursions into biology in the context of pattern formation as well. Parts can be used in teaching: the chapters on multifractals are excellent and include methods to measure fractal dimensions and applications; the discussion of chaos—mostly in the context of one- and two-dimensional maps—is self-contained and comes accompanied and helped by heavy doses of symbolic dynamics.

The first sentence of the "Preface" defines the territory: "The unifying concept underlying fractals, chaos, and power laws is self-similarity." Under this umbrella, an enormous range is covered: exact and statistical self-similarity, reaching into Bethe trees, fractals and multifractals and including along the way a whole array of physical examples, and unsuspected uses of self-similarity, such as two beautiful graphical proofs of

$$1+r+r^2+\cdots=1/(1-r) \text{ for } |r|<1 \quad \text{(pp. 93–94).}$$

There are also discussions of normal numbers, the computation of π, and a discussion of whether or not π is normal (a normal number being a number, in which every possible block of digits in a given numerical system is equally likely to occur). The book makes as strong a case as I have ever seen for the usefulness of binary notation and diophantine expansions. The meaning of the golden mean is made transparent, and chaos in the bakers transformation becomes easily understood. Things are routinely explained in novel ways: a fictitious Sir Pinski's game to give physical meaning to a fractal in terms of initial conditions of a map (the Sierpinski gasket); self-similarity arising from paper folding; self-similarity and possibly the seeds or renormalization in terms of the placement of nonattacking queens in a chessboard; and mode-locking in terms of the frustrated Manhattan pedestrian. Every chemical engineer will enjoy the compact account of the consequences of similarity in systems with homogeneous potentials described in pp. 66–68. I am sure that no two readers will agree on the choice of their three most favorite illustrations. Of engineering *per se* there is little; the closest that the book becomes to actual engineering is in the design of concert halls and a self-similar antenna capable of capturing a wide spectrum of wavelengths. There are, however, plenty of examples of power laws and scaling that can be used for illustrative purposes.

The fact that the book is beautifully written and that covers cutting edge developments justifies that it should be read; the fact that it is thought-provoking makes the case more compelling. This, however, is clearly not going to be enough for people interested in hard-core engineering applications. It is unclear what topics might emerge in the future as a standard part of the repertoire of chemical engineering concepts, and this book offers clearly more that will ever be part of a standard course of an undergraduate chemical engineering student in the mandatory undergraduate physics course. It is clear, however, that some topics, such as percolation, the use of fractals as a language, time series analysis, chaotic dynamics, or more precisely, its older support provided by nonlinear dynamics, and possibly quasicrystals, are here to stay. It is hard to predict what will become of self-organized criticality, cellular automata, and wavelets, the last topic being the only hot topic with obvious connection to self-similarity that I could not find in the book. The building of the foundations of engineering science and assembly of courses constituting a discipline are far from being a completely rectilinear affair and involve lots of near-misses and retreats. There is always overshooting, and time has shown that initial euphoria is sometimes regrettably short-lived. Only time will tell whether or not some of these things will become part of the normal paradigm of engineering science. However, the only way to decide if they are useful is to learn them.

It would be easy to focus on missing things and lose sense of the broader picture: the brief section on viscous fingering is simplistic (only one reference), there could be more on DLA including reference to some of the seminal papers; even in percolation, the case can be made that it does not go far enough into some of the most current applications. These would be shortsighted comments, and everyone will have a different list. There was more in acoustics than I could digest, but probably less than someone in acoustics would like.

Most queries are minor: having seen in it G. I. Barenblatt's book *Similarity, Self-Similarity, and Intermediate Asymptotics*, I did not know that the proof of Pythagoras' theorem based on scaling could be attributed to 11-year-old Einstein (Barenblatt quotes A. B. Migdal, and Migdal does not quote anybody else). Sometimes the historical footnotes get a bit too cute (for example, p. 132 on Leibniz, having come up with the idea of grooves for nails). Most others, however, are packed with information. Reading the book sometimes I got the feeling that Dr. Schroeder was feverishly putting on paper everything he knew.

The focus, however, should be on what the book does rather than what it does not. This is an incredibly entertaining book to read packed with gems of information and a quite healthy amount of historical setting that makes refreshing reading. The only thing I regret is that after opening the Preface with Hermman Weyl's quote on symmetry and shortly thereafter stating, in the second paragraph of the book, to be precise, "Symmetry is one of the most fundamental and fruitful concepts of human thought," the word symmetry is conspicuously absent from the index. This is not a terribly important oversight, but it somehow reinforces the point that the idea of symmetry, so forcefully put forward at the beginning of the book is not taken up at the end. The book ends in a *pianissimo* rather than the *allegro* or *presto* I was hoping for. A closing essay might have made the story complete.

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